p_r for some specified compression η, one requires an additional shock condition based on thermodynamic conservation of energy:

$$\Delta e_D = (\tfrac{1}{2}) p_D (v_0 - v), \tag{3}$$

where Δe_D represents the internal energy change required to reach the shock pressure p_D at the final specific volume v. The energy change required to reach this same final volume isentropically may be found by integration:

$$\Delta e_s = \int_v^{v_0} p dv. \tag{4}$$

The energy excess under conditions produced by the shock is thermal energy. This energy difference may be expressed in terms of thermodynamic data by the equation:

$$\Delta e_D - \Delta e_s = \int_{p_s}^{p_D} C_v (\partial T/\partial p)_v dp. \tag{5}$$

Equations (3), (4), and (5) when combined result in an equation from which the difference $p_D - p_s$ may be found. Estimates of the integrand in Eq. (5) are available, and for our immediate purposes, appropriate mean values can be selected with sufficient accuracy. In this case an explicit solution for $(p_D - p_s)$ may be obtained in the simple form:

$$p_D - p_s = \frac{(\tfrac{1}{2}) p_s (v_0 - v) - \int_v^{v_0} p dv}{C_v (\partial T/\partial p)_v - (\tfrac{1}{2})(v_0 - v)}. \tag{6}$$

In order to solve Eq. (6) a preliminary estimate is made for p_s as a function of v at constant entropy, in which case the numerator of Eq. (6) may easily be evaluated. The specified correction may then be applied to the observed shock pressure p_D and a second approximation made for the isentropic pressure-volume relation. Further approximations can readily be made if necessary.

The initial free surface velocity of the target plates with which we have experimented is approximately twice the mass velocity. More precisely, however, the excess σ of the free surface velocity over mass velocity is given by the Riemann velocity

$$\sigma = \int_{\rho D}^{\rho 1} (c/\rho) d\rho \tag{7}$$

an expression which may be derived on the assumption that the material compressed by the shock expands isentropically to density ρ_1 when traversed by a wave of rarefaction. The difficulty involved in evaluating σ stems from the fact that the entropy of the expanding material, though constant, is different from the original entropy, and accordingly, to determine the isentropic equation of state, the experimental data require correction. Before considering the corrections necessary it is convenient to transform Eq. (7) to the form

$$\sigma = \int_0^{p_D} (-\partial v/\partial p)_s^{\frac{1}{2}} dp. \tag{8}$$

Evaluation of σ as specified in Eq. (8) may be facilitated by the following artifice. First eliminate the shock velocity D between Eqs. (1) and (2) obtaining an expression for the mass velocity u in the form

$$u = \int_0^{p_D} \left(\frac{v_0 - v}{p_D}\right)^{\frac{1}{2}} dp, \tag{9}$$

the integrand of which is constant once a particular volume v corresponding to the extreme pressure has been chosen. Combining Eqs. (8) and (9) there results

$$(\sigma - u)/u = (1/p_D) \int_v^{v_1} \left\{ 1 - \left[(\partial v/\partial p)_s \frac{p_D}{v - v_0} \right]^{\frac{1}{2}} \right\} \left(\frac{\partial p}{\partial v} \right)_s dv, \tag{10}$$

where v_1 is the final specific volume after the isentropic expansion. Equation (10) clearly implies that for sufficiently weak shock the free surface velocity approaches twice the mass velocity. Furthermore, approximate values of the integrand of Eq. (10) suffice to provide an estimate of the difference between u and σ.

In order to determine $(\partial p/\partial v)_s$ at the entropy of the shocked material, one may compute the entropy change Δs due to the shock, then estimate from available thermodynamic data the magnitude of $\partial^2 p/\partial s \partial v$, and thus obtain a corrected value of the desired derivative in the form

$$(\partial p/\partial v)_s = (\partial p/\partial v)_{s0} + (\partial^2 p/\partial s \partial v) \Delta s. \tag{11}$$

A formula for $\partial^2 p/\partial s \partial v$ in terms of readily available thermodynamic data is

$$\frac{\partial^2 p}{\partial s \partial v} = -\frac{T}{C_v^2}\left(\frac{\partial p}{\partial T}\right)_v^2 + \frac{T}{C_v}\frac{\partial^2 p}{\partial T \partial v} + \frac{T^2}{C_v^3}\left(\frac{\partial p}{\partial T}\right)_v^2 \left(\frac{\partial C_v}{\partial T}\right)_v - \frac{2T^2}{C_v^2}\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial^2 p}{\partial T^2}\right)_v \tag{12}$$

in which the first term on the right is the most important.

The entropy excess Δs of the shocked material may be computed by integrating with respect to temperature at the final known volume of the compressed material.

The first step in this process is to determine the isentropic temperature T_s at the compressed volume v from the formula

$$\ln(T_s/T_0) = \int_v^{v_0} (1/C_v)(\partial p/\partial T)_v dv. \tag{13}$$